National Aeronautics and Space Administration
Goddard Space Flight Center
Contract No. NAS-5-3760

ST — TE — 10322

NASA TT F-9673

LOSS OF STABILITY BY ROTATIONAL SHELLS IN TORSION

by

A. V. Pogorelov

[USSR]

3 MAY 1965

NASA TT F-9673
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Doklady A. N. SSSR,
Teoriya uprugosti,
Tom 160, No. 6, 1283 - 4,
Izdatel'stvo "NAUKA", 1965

by A. V. Pogorelov

SUMMARY

22625

This paper deals with the problem of determining the critical value of the moment M, created by uniformly distributed tangential forces along the edge of a convex shell of rotation, which lead to stability loss.

Author

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Assume that a strictly convex rotational shell is under the action of a moment M, created by uniformly distributed tangential forces along its edge. When this moment reaches a certain critical value, the shell loses its stability and there appears on its surface a system of hollows, regularly distributed along the parallel (Fig. 1). Our problem consists in determining such critical moment M.

Because the load received by the shell at time of loss of stability is stationary, the critical value of M can be determined as the moment received by the shell at notable bulging. The deformation of the shell can then be approximated by the isometric transformation of the original shape, and we may limit ourselves to taking into account only the deformation energy at the boundary of the bulging region, Then, the deformation $\overline{U}$ per unit

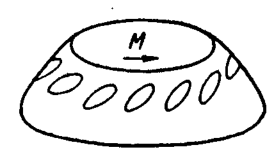

Fig. 1

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of boundary length γ will be determined by the formula

$$\bar{U} = \frac{2E\delta^2\alpha h}{\sqrt{12}\,(1-\nu^2)\,\rho}. \qquad (1)$$

Here α is the angle between the contiguous plane of the curve γ and the tangential planes of the surface; ρ is the curvature radius of γ; h is the variation of the normal camber of the shell at transition through the bulging boundary. The remaining quantities have the usual meaning: E is the elasticity module, ν the Poisson coefficient, δ is the thickness of the shell.

Because at the initial stage of shell's supercritical deformation, it undergoes substantial changes only at bulging region boundaries, the finite bending of the surface of the shell, beyond the small vicinity of the indicated boundary can be substituted by an infinitely small one. The bending fields of infinitely small bendings inside the bulging regions and beyond them are subject at the boundary to a special interlinking condition, naturally stemming from the fact, that the deformation of the the shell is near the isometric transformation. As a result, an obvious analytical expression is obtained for the deformation of the shell in the zone of substantial shape variations. We shall not bring forth this expression, indicating only, that the energy of deformation, determined per unit of boundary length by formula (1), being computed for the entire boundary of all the n regions of bulging, will be

$$U = \frac{2E\delta^2 2\pi}{\sqrt{12}\,(1-\nu^2)\sqrt{R_1R_2}}\,(\lambda^4+\mu^4+4\lambda^2\mu^2)\,\sigma n.$$

Here R_1 and R_2 are the main curvature radii at the centers of bulging regions. The parameters λ and μ characterize the shape of bulging regions and the quantity σ - the value of sagging in the regions. The formula is obtained in the assumption, that the bulging regions are small that they have an elliptical shape and are densely disposed (great n).

Further, we determine the work performed by the moment M

$$A = M\varepsilon,$$

where ε is the angle of torsion of the shell, this quantity being also dependent on the parameters characterizing the bulging regions, and the following formula is obtained for it

$$\varepsilon = \frac{24\pi \sin 2\vartheta\lambda\mu\,(\lambda^2 + \mu^2)\,\sigma}{\rho\Delta y}.$$

Here ϑ is the angle, at which the axes of bulging regions are inclined to the meridian; ρ is the radius of the parallel, over which are disposed the centers of the bulging regions; Δy is the distance between the centers of these regions.

From the condition of shell's equilibrium

$$\frac{d}{d\sigma}(U - A) = 0$$

we obtain the correlation for the moment M, received by the shell at bulging. Bearing in mind that $n\Delta y = 2\pi\rho$, thus correlation can be given the form

$$\frac{\pi\rho^2 E\delta^3}{\sqrt{12(1-\nu^2)}\sqrt{R_1R_2}}\,(1 + 2\omega^2) - 3\sin 2\vartheta\omega M = 0,$$

where $\omega = \frac{\lambda\mu}{\lambda^2 + \mu^2}$. The critical value of M is the least value determined by this correlation. It is obtained at $\omega = ½$ and $\vartheta = 45°$.

Therefore, the critical value of M is determined by the formula

$$M_\kappa = \frac{\pi\rho^3 E\delta^3}{\sqrt{12(1-\nu^2)}\sqrt{R_1R_2}}.$$

The loss of stability is accompanied by the formation on shell's surface of strongly stretched hollows ($\omega = ½$, inclined to the meridian at the angle $\vartheta = 45°$ (Fig. 1).

**** THE END ****

Physico-Technical Institute of
Low Temperatures of the
USSR Academy of Sciences.

Received on 27 Nov. 1964.

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CONTRACT No.NAS-5-3760.
Consultants & Designers, Inc.
Arlington, Virginia

Translated by ANDRE L. BRICHANT
on 3 May 1965

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